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Argonne National Laboratory

PERFORMANCE STATISTICS FOR THE
FORTRAN IV (H) AND
PL/I (VERSION 5) LIBRARIES
IN IBM OS/360 RELEASE 18

by

Kenneth E. Hillstrom

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Applied Mathematics Division

August 1970

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PERFORMANCE STATISTICS FOR THE FORTRAN IV (H)
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ABSTRACT

The computational subroutine libraries associated with the FORTRAN IV (H) and PL/I (F) (Version 5) compilers of OS Release 18 have been tested. The testing techniques are described and accuracy and timing statistics are presented; proper operation under error conditions was verified.

I. INTRODUCTION

The software component of the IBM System/360 Model 50 computer, delivered to Argonne National Laboratory in 1966, included the original FORTRAN IV (H) library. An independent certification of the library undertaken at the Laboratory, with results published in ANL-7321 (see Ref. [1]), detected certain deficiencies in some of its members.

When IBM decided to proceed with the floating point engineering changes, the group at the University of Chicago which had written the original library was asked to modify that library to take advantage of the hardware changes and to eliminate many of the known deficiencies. This reworked library was made available to IBM 360 users with OS Release 18. The present report contains the results of applying the same tests reported in ANL-7321 to this new FORTRAN IV (H) library and of applying similar tests to the PL/I (F) (Version 5) library.

Although results indicate that, in general, both the new libraries are definite improvements over previous versions, some weak members remain.

II. TESTING TECHNIQUES

All certification runs were made under the standard operating system OS/360 (MVT) on a model 50-75 computer. A thorough discussion of the development of the techniques employed in these certifications is presented in Ref. [3].

The tests applied are of three types as follows:

1. Accuracy Tests - These procedures were by far the most extensive. For example, a test of a single-precision real function consisted of the following steps. The domain of definition of the function was divided into several subdomains, usually reflecting the algorithm employed. Next, several thousand uniformly distributed pseudorandom, single-precision, floating-point numbers, together with their identically valued double-precision counterparts, were generated for each subdomain. The single- and

double-precision numbers were used as arguments for the routine tested and for its double-precision counterpart, respectively. The single-precision results $F(x)$ were converted to double precision and the double-precision results $f(x)$ were used as the standard of comparison.

The maximum relative error

$$M = \max \left| \frac{F(x) - f(x)}{f(x)} \right|$$

and the root-mean-square of the relative error

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\frac{F(x_i) - f(x_i)}{f(x_i)} \right)^2}$$

were computed in double precision. Finally, the double-precision results, correctly rounded to single precision, were used as comparison standards for the single-precision results. The distribution of errors in the low order bits of the mantissa was tabulated.

The single-precision complex function tests required the above procedures for both the real and imaginary components. In addition, tests of the complex functions in the PL/I library computed the relative error and root-mean-square of the relative error for the complex function values.

The double-precision function tests necessitated the generation of higher precision comparison values. Since this is not convenient on System/360 machines, the Control Data 3600 computer, with its larger word length, was used to obtain these values. This was accomplished by generating pseudo-random double-precision floating point numbers in CDC 3600 format (85-bit mantissa). These numbers were then converted to a mantissa length (56-bit including possibly up to three leading zeros due to normalization in base 16 arithmetic) compatible with the System/360 double-precision arithmetic by rounding and zeroing out the extra bits. These arguments were then used to generate the corresponding function values on the CDC 3600, the results were rounded, and both the arguments and results were translated into correct System/360 format. Finally the pairs of translated arguments and function values were transmitted to the System/360 machine via magnetic tape, and the testing was carried on there in a manner analogous to the single-precision testing, except that all computations were carried out in double precision.

The accuracy test procedures applied to the FORTRAN-IV and PL/I libraries are written in FORTRAN and are similar in structure. However, the PL/I drivers require a special interface and several PL/I-coded defining routines (PL1CAL and DUMRS, DUMRL, DUMCS, DUMCL) in order to obtain PL/I library members of the required type and precision. (See Ref. [2] for details concerning PL1CAL and Appendix D for an example using PL1CAL and one of the defining routines.)

2. Timing Tests - The timing tests were run under MVT with a storage requirement large enough to insure sole occupancy in the system (800K under our configuration). Also, a 30-second delay was executed immediately preceding the timing tests in order to prevent interruption due to input/output

buffering. These timing procedures called the routine tested $N = 5,000$ or 10,000 times, with either fixed, random, or uniformly distributed arguments, in a loop. The loop "overhead" time was obtained by repeating the same loop using a dummy subroutine whose only executable instruction was a return to the calling program. The routine's execution time was obtained by subtracting the overhead loop time from the test loop time and dividing by N . Due to the coarseness of the internal clock the test loop timings for a particular routine may vary by $\pm 1/60$ second between runs. Therefore, the test timings should not be considered as precise values.

3. Error Return Tests - The error return tests supplied arguments outside the domains of the routines tested. Without exception, the FORTRAN IV tests caused the extended error handling procedure to print the appropriate message and perform the standard fixup action.

The PL/I tests resulted in the execution of the $\emptyset N$ ERR $\emptyset R$ instruction in the PL/I test driver, the printing of an identifying message, and the return of control to the test routine.

III. RESULTS

Appendices A and B are tabular summaries of the FORTRAN IV and PL/I library tests. The FORTRAN IV test results are arranged in alphabetical order according to the entry points of the routines tested. The PL/I results are arranged in alphabetical groups according to the entry points. The intragroup order is (real single, real double, complex single, complex double).

The tabular summaries are interpreted as follows: A typical error distribution heading might be

LE-5	-4BT	-3BT	-3	-2	-1	0	1	2	3	3BT	4BT	GE+5
------	------	------	----	----	----	---	---	---	---	-----	-----	------

Column 0 contains the frequency of exact agreement with the standard. Columns 1, 2, 3 and -1, -2, -3 contain the frequencies of values that are 1, 2, 3 units too high or too low, respectively. Columns 3BT, 4BT and -3BT, -4BT similarly contain the frequencies of 3 bit (4-7 unit) and 4 bit (8-15 unit) disagreements and columns GE+5 and LE-5 contain the frequencies of >5 bit (>16 unit) disagreements.

In addition, the M.R.E. and R.M.S. columns contain the maximum relative errors and the root-mean-square relative errors of the real functions or the complex components. The T.M.R.E. and T.R.M.S. columns of the PL/I tables contain similar information about the complex functions.

Seeming discrepancies in the error tables occur when a test with perhaps 4-bit errors has a smaller maximum relative error than a test with perhaps only 2-bit errors. This is due to the variation in the number of significant bits in the mantissa of a normalized floating point number in base 16 representation. In single precision, a number may contain as many as 24 significant bits in the mantissa (no leading zero bits) or as few as 21 (three leading zero bits). The frequency of error distribution table measures the error in units of the mantissa while the relative

error measures in terms of the number of correct significant digits. A 24-bit number with the last 4 bits in error can still contain more correct significant bits than a 21-bit number with only the last 2 bits in error, hence the 24-bit number would have a smaller relative error.

Note that the same number of random arguments were used in each subinterval for testing a particular function, allowing a direct comparison of the performance of a given routine for various intervals.

To aid in estimating the number of correct significant bits corresponding to a stated maximum relative error, Appendix C tabulates the decimal equivalents of selected inverse powers of 2.

Typical accuracy test programs for the FORTRAN IV and PL/I libraries are listed in Appendix D.

APPENDIX A

Summary of Results - FORTRAN IV (H) Library Tests

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

ALGAMA LOGARITHM OF THE GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS									MAX REL ERROR	ROOT MEAN SQUARE
	ERROR IN THE LAST HEXADECIMAL DIGIT										
0	1	2	3	3BTS	4BTS	5BTS	OTHER				
(0.0,1.0)	430	393	121	110	396	366	13	171		.245E-02	.626E-04
(1.0,2.0)	843	604	42	36	134	207	110	24		.196E-03	.730E-05
(2.0,8.0)	469	988	160	41	182	124	0	36		.539E-02	.121E-03
(8.0,12.)	334	534	379	364	389	0	0	0		.117E-05	.291E-06
(12., -)	760	963	243	34	0	0	0	0		.997E-06	.305E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0,2.0)	104.0
RANDCM IN (2.0,8.0)	121.0
RANDCM IN (8.0, -)	91.0
X=0.5	101.0
X=1.5	100.0
X=10.0	93.0

ERROR RETURN IHC291I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

ALOG NATURAL LOGARITHM

RANGE	ACCURACY FOR RANDOM ARGUMENTS						MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2			
(0.0 ,0.0625)	0	12	1662	326	0		.249E-06	.955E-07
(0.0625,0.125)	0	0	1459	541	0		.367E-06	.161E-06
(0.125 ,0.25)	0	0	1328	672	0		.594E-06	.260E-06
(0.25 ,0.5)	101	719	799	381	0		.872E-06	.287E-06
(0.5 ,0.6065)	0	543	1336	121	0		.122E-06	.464E-07
(0.6065,0.779)	0	242	1470	288	0		.249E-06	.728E-07
(0.779 ,0.8825)	0	211	1462	327	0		.462E-06	.142E-06
(0.8825,0.9395)	0	205	1452	343	0		.912E-06	.282E-06
(0.9395,1.0)	0	322	1412	266	0		.756E-06	.100E-06
(1.0 ,1.065)	0	350	1459	151	0		.738E-06	.996E-07
(1.065 ,1.134)	0	328	1509	163	0		.930E-06	.279E-06
(1.134 ,1.285)	0	375	1477	148	0		.432E-06	.140E-06
(1.285 ,1.65)	0	209	1482	309	0		.211E-06	.683E-07
(1.65 ,2.0)	0	93	1264	643	0		.125E-06	.491E-07
(2.0 ,4.0)	0	548	889	498	65		.843E-06	.323E-06
(4.0 ,8.0)	0	655	1345	0	0		.596E-06	.245E-06
(8.0 ,16.0)	0	540	1460	0	0		.359E-06	.154E-06
(16.0 ,256.0)	0	135	1861	4	0		.227E-06	.651E-07

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (1.0,10.)	47.0

ERROR RETURN IHC253I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

ARCOS ARCCOSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS			MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2		
(0.0, 1.0)	6206	3794	0	.949E-06	.307E-06
(-1.0,-0.5)	1615	285	0	.603E-06	.168E-06
(-0.5, 0.0)	406	1594	0	.776E-06	.450E-06
(0.0, 0.5)	1060	540	0	.877E-06	.411E-06
(0.5,0.84)	1338	662	0	.924E-06	.184E-06
(0.84, 1.0)	1450	550	0	.709E-06	.993E-07

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0,1.0)	59.0
X=-0.8	87.0
X=-0.3	43.0
X= 0.3	37.0
X= 0.7	81.0
X= 0.9	84.0

ERROR RETURN IHC257I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

ARSIN ARCSINE*

RANGE	ACCURACY FOR RANDOM ARGUMENTS			MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2		
(0.0, 1.0)	7430	2570	0	.909E-06	.197E-06
(-1.0,-0.5)	1291	709	0	.917E-06	.254E-06
(-0.5, 0.0)	1710	290	0	.540E-06	.102E-06
(0.0, 0.5)	1679	321	0	.510E-06	.100E-06
(0.5,0.84)	1402	598	0	.126E-06	.385E-07
(0.84, 1.0)	1062	938	0	.934E-06	.454E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0,1.0)	60.0
X=-0.8	86.0
X=-0.3	39.0
X= 0.3	35.0
X= 0.7	83.0
X= 0.9	83.0

ERROR RETURN IHC257I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY
ATAN ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	-5	-4	-3	-2	-1	0	1	2	3	4		
(0.0 , .26795)	0	0	0	1	2056	6620	905	312	100	6	.836E-06	.187E-06
(.26795 , .4142)	0	0	0	106	1632	5071	3001	190	0	0	.527E-06	.132E-06
(.4142 , 1.0)	0	0	0	21	2396	5656	1817	110	0	0	.288E-06	.662E-07
(1.0 , 2.4142)	24	524	2160	1174	4124	1994	0	0	0	0	.108E-05	.511E-06
(2.4142 , 3.732)	0	0	0	0	6503	3497	0	0	0	0	.994E-06	.547E-06
(3.732 , 10.0)	0	0	0	0	7518	2482	0	0	0	0	.949E-06	.546E-06
(10.0 , 3076.0)	0	0	0	0	7612	2388	0	0	0	0	.785E-06	.498E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0,1.0)	37.0
X=0.1340	42.0
X=0.3411	54.0
X=0.7071	52.0
X=1.707	59.0
X=3.073	57.0
X=6.866	47.0
X=1543.0	50.0

NC ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CABS COMPLEX ABSOLUTE VALUE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	-3	-2	-1	0	1	OTHER		
THETA(0,2PI)R(0,1)	9	295	661	727	103	5	.857E-06	.196E-06
THETA(0,2PI)R(0,-)	2	145	678	900	71	4	.924E-06	.295E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0,2PI)(0,1)	70.0
RANDCM IN (0,2PI)(1,-)	70.0

NC ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CCOS COMPLEX COSINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	0	1	2	3	3BTS	4BTS	5BTS	MAX REL ERROR	ROOT MEAN SQUARE
X(0.0,.35)	1661	2457	380	321	164	16	1	.190E-05	.346E-06
Y(0.0, -)	402	1631	886	641	1217	206	17	.222E-05	.622E-06
X(.35, -)	1110	2391	561	333	492	107	6	.198E-05	.450E-06
Y(0.0, -)	1112	2399	547	363	505	73	1	.209E-05	.435E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN X(0,.35)Y(0,-)	193.0
RANDCM IN X(.35,-)Y(0,-)	194.0

ERRCR RETURN IHC275I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CDABS COMPLEX DOUBLE ABSOLUTE VALUE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	-3	-2	-1	0	1	OTHER	MAX REL ERROR	ROOT MEAN SQUARE
THETA(0,2PI)R(0,1)	11	298	903	678	100	10	.221E-15	.516E-16
THETA(0,2PI)R(0,-)	2	160	950	871	17	0	.221E-15	.991E-16

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0,2PI)(0,1)	85.0
RANDCM IN (0,2PI)(1,-)	95.0

NC ERROR RETURN

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

CCOS COMPLEX DOUBLE COSINE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER		
X(0., .35)	443	948	261	56	211	80	1	0	.440E-15	.120E-15
Y(0., -)	117	222	193	170	397	399	248	254	.259E-11	.740E-13
X(.35, -)	82	165	114	87	223	268	256	805	.103E-11	.461E-13
Y(0., -)	95	160	142	90	231	239	248	795	.689E-11	.188E-12

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

RANDCM IN X(0.,.35)	290.0
RANDCM IN X(.35,-)Y(0,-)	286.0

ERRCR RETURN IHC285I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

CCEXP COMPLEX DOUBLE EXPONENTIAL

RANGE	ACCURACY FOR RANDOM CCMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE		
	0	1	2	3	3BT	4BT	5BT	6BT				
X(0,-)	382	666	256	120	226	159	101	47	11	32	.466E-12	.113E-13
Y(0,PI)	448	814	245	128	239	75	25	14	4	8	.817E-13	.192E-14
X(0,-)	370	547	208	111	219	173	132	99	58	83	.118E-11	.293E-13
Y(PI,-)	308	514	200	130	222	192	154	113	67	100	.352E-11	.818E-13

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

RANDCM IN X(0,-)Y(0,PI)	270.0
RANDCM IN X(0,-)Y(PI,-)	270.0

ERRCR RETURN IHC281I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CDLOG COMPLEX DOUBLE EXPONENTIAL

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	6BT	7BT	OTHER		
X(-,+)	430	716	249	133	320	88	13	11	31	9	.563E-13	.227E-14
Y(-,1)	997	847	112	13	30	1	0	0	0	0	.386E-15	.802E-16
X(-,+)	394	848	151	77	334	172	1	2	10	11	.182E-13	.647E-15
Y(-,+)	751	1125	78	22	24	0	0	0	0	0	.424E-15	.102E-15

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN X(-,+)	Y(-,1) 215.0
RANDCM IN X(-,+)	Y(-,+) 230.0

ERRCR RETURN IHC283I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CCSIN COMPLEX DOUBLE SINE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER				
X(0.,.35)	41	483	388	272	615	174	26	1			.706E-15	.196E-15
Y(0., -)	502	1009	142	64	207	76	0	0			.435E-15	.110E-15
X(.35, -)	119	239	136	90	269	250	260	637			.378E-11	.110E-12
Y(0., -)	121	206	127	106	255	238	269	678			.832E-12	.339E-13

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN X(0.,.35)	Y(0,-) 287.0
RANDCM IN X(.35,-)	Y(0,-) 287.0

ERRCR RETURN IHC285I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CDSQRT COMPLEX DOUBLE SQUARE ROOT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	-4BT	-3BT	-2	-1	0	1	2	3	3BT	4BT	MAX REL ERROR	ROOT MEAN SQUARE	
X(-1,1)	3	86	33	72	684	1047	66	9	0	0	.221E-15	.507E-16	
Y(-1,1)	3	44	10	37	390	1071	359	46	12	28	0	.218E-15	.465E-16
X(-, +)	30	147	46	86	445	1055	101	35	23	32	0	.246E-15	.760E-16
Y(-, +)	14	81	36	62	296	1047	254	64	42	92	12	.222E-15	.733E-16

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN X(0,2PI)Y(0,1)	160.0
RANDCM IN X(0,2PI)Y(1,-)	170.0

NO ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CEXP COMPLEX EXPONENTIAL

ACCURACY FOR RANDOM COMPLEX ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	0	1	2	3	3BTs	4BTs	5BTs	6BTs	MAX REL ERROR	ROOT MEAN SQUARE
X(0,-)	774	866	140	81	125	13	1	0	.216E-05	.360E-06
Y(0,PI)	719	924	140	75	125	16	1	0	.152E-05	.358E-06
X(0,-)	741	888	125	86	145	13	2	0	.167E-05	.368E-06
Y(PI,-)	727	890	143	94	131	14	1	0	.190E-05	.368E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN X(0,-)Y(0,PI)	165.0
RANDCM IN X(0,-)Y(PI,-)	175.0

ERRCR RETURN IHC271I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

CLOG COMPLEX LOGARITHM

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	7BTS	OTHER		
X(-,+)	617	633	167	141	314	69	6	17	30	6	.170E-03	.636E-05
Y(-,+1)	1028	839	60	12	43	18	0	0	0	0	.139E-05	.354E-06
X(-,+)	675	643	105	115	365	73	3	3	11	7	.617E-04	.199E-05
Y(-,+)	654	1175	75	14	58	24	0	0	0	0	.124E-05	.485E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN X(-,+)*Y(-,1)	150.0
RANDCM IN X(-,+)*Y(-,+)	160.0

ERRCR RETURN IHC273I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

COS COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS			
(0.0,PI/2)	1810	3006	141	1	14	28	144E-05	.192E-06	
(PI/2,PI)	1757	3044	153	3	16	27	.333E-05	.211E-06	
(PI ,2PI)	1801	3008	147	4	17	23	.143E-05	.199E-06	
(2PI,8PI)	1826	3007	124	2	22	19	.148E-05	.190E-06	
(8PI,64PI)	1814	3001	139	3	13	30	.147E-05	.191E-06	

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0,PI/2)	41.0
X=0.7854	40.0
X=5.0	45.0
X=20.0	46.0

ERRCR RETURN IHC254I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

COSH HYPERBOLIC COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(0.0 ,0.35)	0	717	2142	141	0	.107E-05	.419E-06
(0.35,12.9)	2	615	1975	408	0	.114E-05	.222E-06
(12.9, -)	0	59	2073	868	0	.107E-05	.202E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0 ,0.35)	85.0
RANDCM IN (0.35,12.9)	88.0
RANDCM IN (12.9, -)	89.0
X = 0.20	87.0
X = 8.94	90.0
X = 100.0	88.0

ERRCR RETURN IHC256I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

COT COTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERRCR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS		
(0.0,PI/4)	1815	2666	219	107	169	24	.115E-05	.372E-06
(PI/4,8PI)	1891	2531	414	73	74	17	.242E-05	.316E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0,PI/4)	61.0
X=0.10	50.0
X=0.50	60.0
X=2.00	55.0
X=10.0	60.0

ERRCR RETURN IHC258I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CSIN COMPLEX SINE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS		
X(0.0,.35)	395	1621	881	622	1230	232	19	.222E-05	.634E-06
Y(0.0, -)	1695	2469	327	330	165	14	0	.149E-05	.328E-06
X(.35, -)	1088	2387	562	357	513	91	2	.203E-05	.448E-06
Y(0.0, -)	1131	2399	541	342	491	91	5	.192E-05	.437E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
 RANDCM IN X(0,.35)Y(0,-) 189.0
 RANDCM IN X(.35,-)Y(0,-) 191.0

NO ERROR RETURN IHC275I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CSQRT COMPLEX SQUARE ROOT

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	CTHIR	-3BT	-2BT	-1	0	1	2BT		
X(-1,+1)	4	92	82	666	1069	80	7	0	.889E-06
Y(-1,+1)	4	40	50	363	1072	353	60	53	.923E-06
X(--,+-)	34	200	118	521	1001	84	23	14	.106E-05
Y(--,+-)	13	115	81	269	1020	295	82	111	.104E-05

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
 RANDCM IN X(0,2PI)Y(0,1) 120.0
 RANDCM IN X(0,2PI)Y(1,-) 125.0

NO ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DCOS DOUBLE ARCCOSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(-0.5,0.5)	0	549	1451	0	0	.211E-15	.123E-15
+-(0.5,1.)	241	376	587	13	0	.222E-15	.100E-15

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (-.5,.5)	75.0
RANDCM IN +-(.5,1)	128.0

ERRCR RETURN IHC267I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DCSIN DOUBLE ARCSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(-0.5,0.5)	0	517	975	508	0	.222E-15	.786E-16
+-(0.5,1.)	0	398	1260	341	1	.222E-15	.101E-15

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (-.5,.5)	71.0
RANDCM IN +-(.5,1)	128.0

ERRCR RETURN IHC267I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

DATAN DOUBLE ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	-3	-2	-1	0	1	2	3		
(0.0 , .26795)	0	0	2178	2822	0	0	0	.222E-15	.638E-16
(.26795,.4142)	0	0	52	1609	2668	668	3	.148E-15	.453E-16
(.4142 ,1.0)	4	647	2831	1052	416	50	0	.706E-16	.252E-16
(1.0 ,2.4142)	0	0	1723	1850	1086	340	1	.222E-15	.118E-15
(2.4142,3.732)	0	0	2875	2125	0	0	0	.188E-15	.135E-15
(3.732 ,10.0)	0	0	2146	2854	0	0	0	.170E-15	.103E-15
(10.0 ,3076.0)	0	0	2615	2385	0	0	0	.149E-15	.102E-15

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0,.26795)	63.0
RANDCM IN (.26795,1)	77.0
X=0.50	85.0
RANDCM IN (1.,3.732)	85.0
RANDCM IN (3.732,10)	70.0
RANDCM IN (10,3076.)	72.0

NC ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DCOS DOUBLE COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS		
(-PI/2,PI/2)	82	244	205	117	187	133	1	.173E-13	.107E-14
(PI/2, -)	85	202	114	93	271	129	21	.224E-12	.846E-14

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (-PI/2,PI/2)	79.0
RANDCM IN (PI/2, -)	78.0

ERRCR RETURN INC264I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

DCOSH DOUBLE HYPERBOLIC COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(0.0,0.35)	0	416	584	0	0	.222E-15	.140E-15
(0.35,19.3)	22	498	358	77	5	.425E-15	.902E-16
(19.3, -)	21	471	425	79	4	.420E-15	.861E-16

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0,0.35)	115.0
RANDCM IN (0.35,19.3)	117.0
RANDCM IN (19.3, -)	115.0
x = 0.20	116.0
x = 10.0	118.0
x = 50.0	113.0

ERRCR RETURN IMC266I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

DCOTAN DOUBLE COTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS		
(0.0,PI/4)	1707	2777	268	110	129	9	0	0
(PI/4,8PI)	292	547	387	221	658	743	436	1716

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (PI/4,8PI)	74.0
x=0.10	70.0
x=0.50	70.0
x=2.00	65.0
x=10.0	70.0

ERRCR RETURN IMC268I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DERF DOUBLE ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT			MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2		
(0.0 , 1.317)	1138	836	26	.217E-15	.278E-16
(1.317 , 2.04)	1097	903	0	.148E-16	.953E-17
(2.04 , 6.092)	1050	950	0	.139E-16	.967E-17

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X=0.80	97.0
X=1.60	128.0
X=3.20	197.0
X=6.40	23.0

NO ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DERFC DOUBLE COMPLEMENTARY ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	7BTS	OTHER		
(-6,-1.3)	1075	925	0	0	0	0	0	0	0	0	.114E-15	.756E-16
(-1.3,0.)	1067	933	0	0	0	0	0	0	0	0	.222E-15	.101E-15
(0.0,1.3)	1128	846	26	0	0	0	0	0	0	0	.354E-15	.646E-16
(1.3,2.0)	761	978	181	50	30	0	0	0	0	0	.431E-15	.726E-16
(2.0,3.9)	533	668	303	203	265	24	0	0	0	0	.349E-15	.923E-16
(3.9,13.)	38	96	58	46	192	371	408	367	303	121	.333E-14	.182E-14

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X=-0.80	105.0
X= 0.80	99.0
X= 1.60	130.0
X= 3.20	198.0
X= 14.3	24.9

NO ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

CEXP DOUBLE EXPONENTIAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(0.0 ,0.332)	0	877	1123	0	0	.222E-15	.125E-15
(0.332 ,0.664)	0	899	1101	0	0	.159E-15	.917E-16
(0.664 ,1.328)	0	788	1212	0	0	.114E-15	.533E-16
(1.328 ,2.656)	0	525	1374	101	0	.588E-16	.189E-16
(2.656 ,5.313)	0	659	1255	86	0	.222E-15	.613E-16
(5.313 ,10.625)	0	658	1247	95	0	.222E-15	.614E-16
(10.625,21.250)	0	675	1252	73	0	.222E-15	.618E-16
(21.250,42.500)	0	623	1292	85	0	.222E-15	.594E-16
(42.500,85.000)	0	665	1249	86	0	.221E-15	.615E-16
(85.000,170.00)	0	616	1299	85	0	.222E-15	.623E-16
(0.0 ,-0.332)	0	405	1214	381	0	.193E-16	.102E-16
(-0.332,-0.664)	0	297	1467	236	0	.270E-16	.120E-16
(-0.664,-1.328)	0	537	1389	74	0	.524E-16	.210E-16
(-1.328,-2.656)	0	794	1204	2	0	.197E-15	.760E-16
(-2.656,-5.313)	0	621	1301	78	0	.222E-15	.548E-16
(-5.313,-10.62)	0	588	1306	106	0	.222E-15	.595E-16
(-10.62,-21.25)	0	598	1289	112	1	.221E-15	.582E-16
(-21.25,-42.50)	0	634	1256	109	1	.222E-15	.640E-16
(-42.50,-85.00)	0	620	1269	110	1	.222E-15	.616E-16
(-85.00,-170.0)	0	616	1275	109	0	.222E-15	.581E-16

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
RANCCM IN (-85,-170) 75.0

ERRCR RETURN IHC262I TESTED

TEST OF IBM 360 FCRTAN-IV SUBROUTINE LIBRARY

DGAMMA DCUBLE GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	6BTS	7BTS	OTHER		
(0.0,1.0)	1048	950	2	0	0	0	0	0	0	0	.222E-15	.976E-16
(1.0,2.0)	1538	461	1	0	0	0	0	0	0	0	.283E-16	.723E-17
(2.0,4.0)	663	1262	75	0	0	0	0	0	0	0	.221E-15	.986E-16
(4.0,8.0)	29	470	523	233	563	164	18	0	0	0	.532E-15	.194E-15
(8.0,16.)	43	74	90	94	345	383	370	285	211	105	.598E-14	.180E-14
(16,57.6)	10	24	27	24	92	193	352	438	395	445	.116E-13	.413E-14

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
X=0.06 115.0
X=1.30 115.0
X=7.90 159.0
X=13.7 235.0

ERRCR RETURN IHC300I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DLGAMA DOUBLE LOGARITHM OF THE GAMMA FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER		
(0.0,0.5)	281	551	180	125	425	278	21	139	.287E-12	.734E-14
(0.5,1.5)	164	896	475	54	168	181	35	27	.105E-12	.393E-14
(1.5,4.0)	263	1022	339	47	167	115	5	42	.324E-13	.110E-14
(4.0,12.)	385	481	378	332	397	21	5	0	.419E-15	.746E-15
(12.0, -)	775	853	330	36	2	4	0	0	.402E-15	.716E-15

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0, 0.5)	199.0
RANDCM IN (0.5, 1.5)	194.0
RANDCM IN (1.5, 4.0)	224.0
RANDCM IN (8.0,12.0)	157.0
RANDCM IN (12.0, -)	159.0
X = 0.30	206.0
X = 1.10	196.0
X = 1.70	196.0
X = 11.0	157.0
X = 47.0	157.0

ERROR RETURN IHC301I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DLOG DOUBLE LOGARITHM

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	-3	-2	-1	0	1	2	3	4		
(0.177 ,0.354)	0	0	0	656	1344	0	0	0	.213E-15	.139E-15
(0.354 ,0.707)	300	674	489	483	54	0	0	0	.221E-15	.586E-16
(0.707 ,1.000)	0	0	0	317	1455	228	0	0	.429E-15	.101E-15
(1.000 ,1.414)	0	302	1488	210	0	0	0	0	.442E-15	.995E-16
(1.414 ,2.828)	0	0	73	695	568	425	239	0	.222E-15	.543E-16
(2.828 ,5.657)	0	0	1402	598	0	0	0	0	.213E-15	.132E-15

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (2.8,5.6)	81.0

ERROR RETURN IHC263I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

DPOWER DOUBLE EXPONENTIATION

ACCURACY FOR RANDOM ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	0	1	2B _T	3B _T	4B _T	5B _T	6B _T	7B _T	8B _T	OTHER	MAX REL ERROR	ROOT MEAN SQUARE
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X(2**-4 ,2**4)											.125E-14	.348E-15
Y(-4.0,4.0)	328	461	450	309	220	185	45	2	0	0		
X(2**-16,2**16)											.732E-14	.225E-14
Y(-16.0,16.0)	84	152	169	198	301	355	257	264	162	18		
X(2**-32,2**32)											.505E-13	.954E-14
Y(-8.0,8.0)	109	128	192	168	254	262	230	225	194	238		
X(2**-64,2**64)											.255E-13	.677E-14
Y(-4.0,4.0)	94	121	146	144	167	195	296	312	258	267		
X(2**-8 ,2**8)											.132E-13	.365E-14
Y(-32.0,32.0)	47	100	127	144	214	349	378	328	226	87		
X(2**-4 ,2**4)											.185E-13	.541E-14
Y(-64.0,64.0)	59	108	122	163	241	277	294	295	249	192		

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
X,Y RANDOM IN (0,1) 185.0

ERRCR RETURN IHC244I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

CSIN DOUBLE SINE

ACCURACY FOR RANDOM ARGUMENTS
ERRCR IN THE LAST HEXADECIMAL DIGIT

RANGE	0	1	2	3	3B _T S	4B _T S	5B _T S	OTHER	MAX REL ERROR	ROOT MEAN SQUARE
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(-PI/2,PI/2)	284	497	120	53	38	8	0	0	.385E-15	.513E-16
(PI/2, -)	95	203	133	97	256	121	15	80	.117E-12	.629E-14

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
RANDCM IN (-PI/2,PI/2) 78.0
RANDCM IN (PI/2, -) 77.0

ERRCR RETURN IHC264I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

DSINH DOUBLE HYPERBOLIC SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(0.0,0.35)	0	545	455	0	0	.221E-15	.678E-16
(0.35,19.3)	20	477	426	69	8	.422E-15	.863E-16
(19.3, -)	21	470	425	81	3	.420E-15	.859E-16

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0,0.35)	56.0
RANDCM IN (0.35,19.3)	119.0
RANDCM IN (19.3, -)	117.0
X =0.20	55.0
X =10.0	119.0
X =50.0	114.0

ERRCR RETURN IHC266I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

DSQRT DOUBLE SQUARE ROOT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-1	0	1	2	3		
(0.0, -)	0	1999	1	.291E-16	.650E-18	.291E-16	.650E-18
(.06,1.0)	0	2000	0	.0	.0	.0	.0
(1.0,16.)	0	2000	0	.0	.0	.0	.0

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (.07,5.4)	49.0

ERRCR RETURN IHC261I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DTAN DOUBLE TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS OTHER		
(0.0,PI/4)	1684	2395	764	107	40	10	0	.366E-15	.644E-16
(PI/4,8PI)	307	528	383	235	650	716	387 1794	.421E-11	.956E-13

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDM IN (PI/4,8PI)	70.0
X=0.10	65.0
X=0.50	60.0
X=2.00	70.0
X=10.0	70.0

ERRCR RETURN IHC268I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

DTANH DOUBLE HYPERBOLIC TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT						MAX REL ERROR	ROOT MEAN SQUARE
	-3BT	-3	-2	-1	0	1		
(0.0 ,0.55)	0	0	0	806	1190	4	.215E-15	.487E-16
(0.55,20.1)	9	9	9	20	984	969	.165E-15	.140E-16
(20.1, -)	0	0	0	0	2000	0	.0	.0

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDM IN (0.0 ,0.55)	50.0
RANDM IN (0.55,20.1)	110.0
RANDM IN (20.1, -)	12.0

NC ERROR RETURN

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

ERF ERROR INTEGRAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS			MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2		
(-3.9192,-2.04)	966	1034	0	.597E-07	.346E-07
(-2.04 , -1.317)	1077	923	0	.694E-07	.336E-07
(-1.317 , 0.0)	1164	805	31	.752E-06	.101E-06
(0.0 , 1.317)	1179	787	34	.891E-06	.101E-06
(1.317 , 2.04)	1044	956	0	.684E-07	.340E-07
(2.04 , 3.9192)	955	1045	0	.596E-07	.348E-07

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X = -1.10	64.0
X = 1.10	60.0
X = 1.80	59.0
X = 2.60	107.0
X = 5.10	22.0

NO ERROR RETURN

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

ERFC COMPLIMENTARY ERROR INTEGRAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS	5BTS	OTHER		
(-13.306,-1.317)	1803	197	0	0	0	0	0	0	.453E-06	.119E-06
(-1.317, 0.0)	1116	884	0	0	0	0	0	0	.916E-06	.345E-06
(0.0 , 1.317)	1152	814	34	0	0	0	0	0	.114E-05	.242E-06
(1.317, 2.04)	698	892	323	54	33	0	0	0	.179E-05	.351E-06
(2.04 , 3.9192)	577	625	314	173	256	55	0	0	.149E-05	.369E-06
(3.9192 ,13.306)	35	67	63	55	190	348	415	827	.151E-04	.834E-05

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X=-5.10	26.0
X=-1.10	63.0
X= 1.10	59.0
X= 1.80	55.0
X= 3.80	107.0

NO ERROR RETURN

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY
EXP EXPONENTIAL

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT	ACCURACY FOR RANDOM ARGUMENTS		MAX REL ERROR	ROOT MEAN SQUARE
		0	1		
(0.0 , 0.3466)	1990	10		.473E-06	.233E-06
(0.3466 , 0.6931)	1987	13		.341E-06	.166E-06
(0.6931 , 1.3863)	1982	18		.228E-06	.102E-06
(1.3863 , 2.7726)	1938	62		.125E-06	.401E-07
(2.7726 , 5.5452)	1954	46		.439E-06	.120E-06
(5.5452 , 11.0904)	1969	31		.458E-06	.115E-06
(11.0904 , 22.1807)	1954	46		.455E-06	.118E-06
(22.1807 , 44.3614)	1964	36		.427E-06	.116E-06
(44.3614 , 88.7229)	1949	51		.475E-06	.120E-06
(88.7229 , 174.6730)	1957	43		.440E-06	.117E-06
(-174.6730,-100.0)	1953	47		.470E-06	.114E-06
(-100.0 , -10.0)	1950	50		.460E-06	.114E-06
(-10.0 , -1.0)	1963	37		.458E-06	.116E-06
(-1.0 , 0.0)	1926	74		.790E-07	.308E-07

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
RANDM IN (0.0,1.0) 54.0

ERRCR RETURN IHC252I TESTED

TEST OF IBM 360 FORTRAN-IV SUBROUTINE LIBRARY

GAMMA GAMMA FUNCTION

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT	ACCURACY FOR RANDOM ARGUMENTS						MAX REL ERROR	ROOT MEAN SQUARE	
		0	1	2	3	3BTS	4BTS	5BTS	OTHER	
(0.0, 1.0)	991	1008	1	0	0	0	0	0		.102E-05
(1.0, 2.0)	1335	649	16	0	0	0	0	0		.367E-06
(2.0, 4.0)	589	1257	154	0	0	0	0	0		.115E-06
(4.0, 8.0)	23	421	539	240	554	207	16	0		.923E-06
(8.0,16.0)	39	85	89	90	277	383	409	628		.237E-05
(16.0,57.0)	18	48	43	39	125	276	380	1071		.225E-04
										.432E-04
										.151E-04

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
X= 0.40 54.0
X= 1.10 49.0
X= 2.80 56.0
X= 5.50 75.0
X= 13.7 146.0

ERRCR RETURN IHC290I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY
POWER EXPONENTIATION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT		
X(2**-4 ,2**4)											
Y(-4.0,4.0)	603	414	376	288	200	100	19	0	0	.373E-05	.105E-05
X(2**-16,2**16)											
Y(-16.0,16.0)	156	182	198	220	259	358	286	222	77	.248E-04	.754E-05
X(2**-32,2**32)											
Y(-8.0,8.0)	150	195	183	215	250	285	254	202	129	.115E-03	.216E-04
X(2**-64,2**64)											
Y(-4.0,4.0)	114	164	177	166	201	256	288	297	207	.601E-04	.175E-04
X(2**-8 ,2**8)											
Y(-32.0,32.0)	152	157	215	187	221	331	275	273	161	.369E-04	.108E-04
X(2**-4 ,2**4)											
Y(-64.0,64.0)	166	187	179	189	194	247	310	235	178	.609E-04	.164E-04

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
X,Y RANDOM IN (0,1) 120.0

ERRCR RETURN IHC253I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY
SIN SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERRCR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BTS	4BTS			
(0.0,PI/2)	1853	2985	124	5	16	17	.138E-05	.192E-06	
(PI/2,PI)	1870	2953	131	5	17	24	.213E-05	.187E-06	
(PI ,2PI)	1808	3016	135	3	14	24	.252E-05	.194E-06	
(2PI,8PI)	1772	3073	116	4	14	21	.152E-05	.193E-06	
(8PI,64PI)	1772	3069	115	6	11	27	.140E-05	.185E-06	

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
RANDOM IN (0,PI/2) 44.0
X=0.7854 43.0
X=5.0000 44.0
X=20.000 45.0

ERRCR RETURN IHC254I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

SINH HYPERBOLIC SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(0.0 ,0.35)	0	1291	1709	0	0	.929E-06	.209E-06
(0.35,12.9)	1	549	2032	418	0	.113E-05	.210E-06
(12.9, -)	0	59	2073	868	0	.107E-05	.202E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0 ,0.35)	29.0
RANDCM IN (0.35,12.9)	84.0
RANDCM IN (12.9, -)	91.0
x = 0.20	29.0
x = 8.94	89.0
x = 100.0	89.0

ERRCR RETURN IHC256I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

SQRT SQUARE RCOT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2				
(0.0, -)	9512	488	0			.439E-06	.548E-07
(.06,1.0)	9416	584	0			.113E-06	.304E-07
(1.0,16.)	9874	126	0			.465E-06	.120E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.07,5.40)	34.0

ERRCR RETURN IHC251I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

TAN TANGENT

ACCURACY FOR RANDOM ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	C	1	2	3	3BTS	4BTS	OTHER	MAX REL ERROR	ROOT MEAN SQUARE
(0.0,PI/4)	1904	2482	513	43	3	55	0	.166E-05	.273E-06
(PI/4,8PI)	1877	2599	355	70	83	15	1	.319E-05	.315E-06

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0,PI/4)	55.0
X=0.10	50.0
X=0.50	50.0
X=2.00	50.0
X=10.0	55.0

ERROR RETURN IHC258I TESTED

TEST OF IBM 360 FCRTRAN-IV SUBROUTINE LIBRARY

TANH HYPERBOLIC TANGENT

ACCURACY FOR RANDOM ARGUMENTS
ERROR IN THE LAST HEXADECIMAL DIGIT

RANGE	-3	-2	-1	0	1	2	3	MAX REL ERROR	ROOT MEAN SQUARE
(0.0 ,0.55)	0	0	2100	2899	1	0	0	.863E-06	.166E-06
(0.55,9.01)	1	14	91	2472	2397	25	0	.249E-06	.399E-07
(9.01, -)	0	0	0	5000	0	0	0	.296E-07	.112E-08

AVERAGE EXECUTION TIME ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
RANDCM IN (0.0 ,0.55)	26.0
RANDCM IN (0.55,9.01)	84.0
RANDCM IN (9.01, -)	12.0

NO ERROR RETURN

APPENDIX B

Summary of Results - PL/I Library Tests

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

ALOG NATURAL LOGARITHM

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT					MAX REL ERROR	ROOT MEAN SQUARE
	-2	-1	0	1	2		
(0.0 ,0.0625)	0	19	1651	330	0	.249E-06	.963E-07
(0.0625,0.1250)	0	0	1460	540	0	.367E-06	.161E-06
(0.1250,0.2500)	0	0	1343	657	0	.594E-06	.257E-06
(0.2500,0.5000)	98	728	797	377	0	.872E-06	.287E-06
(0.5000,0.6065)	0	538	1334	128	0	.122E-06	.466E-07
(0.6065,0.7790)	0	231	1479	290	0	.249E-06	.715E-07
(0.7790,0.8825)	0	209	1472	319	0	.462E-06	.140E-06
(0.8825,0.9395)	0	204	1453	343	0	.912E-06	.283E-06
(0.9395,1.0000)	0	319	1421	260	0	.756E-06	.101E-06
(1.0000,1.0650)	0	350	1498	152	0	.738E-06	.997E-07
(1.0650,1.1340)	0	327	1510	163	0	.930E-06	.279E-06
(1.1340,1.2850)	0	374	1477	149	0	.432E-06	.140E-06
(1.2850,1.6500)	0	208	1478	314	0	.211E-06	.685E-07
(1.6500,2.0000)	0	94	1263	643	0	.120E-06	.491E-07
(2.0000,4.0000)	0	552	889	499	60	.843E-06	.324E-06
(4.0000,8.0000)	0	655	1345	0	0	.596E-06	.243E-06
(8.0000,16.0000)	0	543	1457	0	0	.359E-06	.155E-06
(16.0000,256.00)	0	125	1871	4	0	.227E-06	.642E-07

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
UNIFCRM IN (1.0,10.0) 56.0

ERROR RETURN IHE202I FROM IHELNS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DLG NATURAL LOGARITHM

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT				MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3		
(0.177,0.354)	656	1344	0	0	.213E-15	.139E-15
(0.354,0.707)	483	543	674	300	.221E-15	.586E-16
(0.707,1.000)	317	1455	228	0	.429E-15	.101E-15
(1.000,1.414)	210	1468	302	0	.442E-15	.995E-16
(1.414,2.828)	695	641	425	239	.222E-15	.543E-16
(2.828,5.657)	598	1402	0	0	.213E-15	.132E-15

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
UNIFCRM IN (2.828,5.657) 90.0

ERROR RETURN IHE202I FROM IHELNL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CLOG COMPLEX LOGARITHM

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX ERROR	ROOT SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT				
X(--,+)	636	632	173	139	290	67	10	.195E-03	.719E-05		
Y(-1,+1)	1010	847	72	8	49	14	0	.143E-05	.361E-06	.815E-05	.469E-06
X(--,+)	648	670	110	100	370	79	0	.611E-04	.208E-05		
Y(--,+)	640	1183	86	8	59	24	0	.128E-05	.483E-06	.118E-05	.440E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(--,+),	170.0
Y(-1,+1)	
UNIFCRM IN X(--,+),	
Y(--,+)	170.0

ERROR RETURN IHE2061 FROM IHENLN TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDLOG COMPLEX DOUBLE LOGARITHM

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT				
X(--,+)	430	716	249	133	320	88	13	.563E-13	.227E-14		
Y(-1,+1)	1031	815	90	20	42	2	0	.386E-15	.817E-16	.137E-14	.103E-15
X(--,+)	394	848	151	77	334	172	1	.182E-13	.647E-15		
Y(--,+)	776	1111	49	20	44	0	0	.424E-15	.103E-15	.284E-15	.107E-15

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(--,+),	228.0
Y(-1,+1)	
UNIFCRM IN X(--,+),	
Y(--,+)	226.0

ERROR RETURN IHE2061 FROM IHENNZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

ATAN ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	-5	-4	-3	-2	-1	0	1	2	3	4		
(0.0 ,.268)	0	0	0	1	2057	6618	906	313	100	5	.843E-06	.187E-06
(.268,.414)	0	0	0	105	1657	5027	3018	193	0	0	.527E-06	.133E-06
(.414,1.0)	0	0	0	22	2363	5715	1806	94	0	0	.288E-06	.659E-07
(1.0 ,2.41)	24	530	2162	1169	4148	1967	0	0	0	0	.109E-05	.511E-06
(2.41,3.73)	0	0	0	0	6477	3523	0	0	0	0	.994E-06	.547E-06
(3.73,10.0)	0	0	0	0	7524	2476	0	0	0	0	.949E-06	.546E-06
(10.0 ,3076)	0	0	0	0	7606	2394	0	0	0	0	.783E-06	.498E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,1.0)	52.0
X = 0.134	44.0
X = 0.7071	52.0
X = 3.073	58.0
X = 6.866	46.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DATAN DCUBLE ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	-3	-2	-1	0	1	2	3		
(0.0 ,.26795)	0	0	2178	2822	0	0	0	.222E-15	.638E-16
(.26795,0.4142)	0	0	3	524	2690	1701	82	.159E-15	.616E-16
(0.4142,1.0)	0	7	791	3065	866	266	5	.103E-15	.207E-16
(1.0 ,2.4142)	0	10	2264	2332	389	5	0	.222E-15	.123E-15
(2.4142,3.732)	0	0	3018	1982	0	0	0	.188E-15	.138E-15
(3.732 , 10.0)	0	0	2146	2854	0	0	0	.170E-15	.103E-15
(10.0 ,3076.0)	0	0	2615	2985	0	0	0	.149E-15	.102E-15

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,.268)	76.0
UNIFCRM IN (.268,.414)	90.0
UNIFCRM IN (.414, 1.0)	90.0
UNIFCRM IN (1.0,2.41)	96.0
UNIFCRM IN (2.41,3.73)	92.0
UNIFCRM IN (3.73,10.0)	76.0
UNIFCRM IN (10.0,3076)	80.0
X = 0.134	72.0
X = 0.3411	86.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CATAN COMPLEX ARCTANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	C	1	2	3	3BT	4BT	5BT OTHER				
X(-1.,1.)	1270	556	15	18	75	64	2	0	.146E-05	.309E-06	
Y(-9.,9.)	334	824	376	180	217	58	9	2	.178E-04	.837E-06	.168E-04 .690E-06
X(-1.,1.)	1376	596	4	4	10	10	0	0	.116E-05	.259E-06	.292E-05 .268E-06
Y(-80,80)	290	997	286	43	122	224	38	0	.293E-05	.539E-06	
X(-80,80)	1422	574	0	2	2	0	0	0	.714E-06	.254E-06	.711E-06 .254E-06
Y(-9.,9.)	355	932	198	55	174	265	21	0	.210E-05	.565E-06	
X(-80,80)	1424	576	0	0	0	0	0	0	.494E-06	.252E-06	.494E-06 .252E-06
Y(-80,80)	449	1182	175	15	60	117	2	0	.159E-05	.452E-06	

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(-1.,+1.), Y(-9.,+9.)	142.0
UNIFCRM IN X(-80,+80), Y(-80,+80)	142.0

ERRCR RETURN IHE2111 FROM IHEATH TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDATAN COMPLEX DOUBLE ARCTANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	C	1	2	3	3BT	4BT	5BT OTHER				
X(-1.,1.)	1360	492	15	27	71	32	2	1	.131E-14	.858E-16	
Y(-9.,9.)	390	684	392	217	241	50	19	7	.162E-13	.469E-15	.161E-13 .439E-15
X(-80,80)	1493	507	0	0	0	0	0	0	.170E-15	.718E-16	
Y(-80,80)	776	862	139	67	86	59	11	0	.408E-15	.941E-16	.170E-15 .718E-16

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(-1.,+1.), Y(-9.,+9.)	272.0
UNIFCRM IN X(-80,+80), Y(-80,+80)	272.0

ERRCR RETURN IHE2111 FROM IHEATZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

ATANH HYPERBOLIC ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	-3BT	-3	-2	-1	0	1	2	3	3BT		
(0.0,0.2)	0	27	152	737	1084	0	0	0	0	.935E-06	.310E-06
(0.2,0.9)	396	96	144	318	731	240	63	10	2	.106E-05	.298E-06
(-.2,0.0)	0	0	0	0	1094	719	166	21	0	.949E-06	.308E-06
(-.9,-.2)	3	11	53	273	692	326	138	101	403	.953E-06	.310E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,0.2)	38.0
UNIFCRM IN (0.2,0.9)	98.0
UNIFCRM IN (-.2,0.0)	34.0
UNIFCRM IN (-.9,-.2)	94.0

ERROR RETURN IHE208I FROM IHEHTS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DATANH DOUBLE HYPERBOLIC ARCTANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE	
	-4BT	-3BT	-3	-2	-1	0	1	2	3	3BT	4BT		
(0.0,.25)	0	0	0	55	1252	693	0	0	0	0	.222E-15	.767E-16	
(-.25,0.)	0	0	0	0	0	693	1252	55	0	0	.222E-15	.767E-16	
(.25,.95)	136	393	90	99	480	526	220	44	7	5	0	.304E-15	.975E-16
(-.95,-.25)	0	5	7	44	220	526	480	99	90	393	136	.304E-15	.975E-16

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,0.25)	70.0
UNIFCRM IN (-.25,0.0)	64.0
UNIFCRM IN (0.25,0.95)	122.0
UNIFCRM IN (-.95,0.25)	120.0

ERROR RETURN IHE208I FROM IHEHTL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDATANH COMPLEX HYPERBOLIC ARCTANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERRCR IN THE LAST HEXADECIMAL DIGIT								COMPONENT			COMPLEX	
									MAX	ROOT	MAX	ROOT	
	0	1	2	3	3BT	4BT	5BT	OTHER	REL	MEAN	REL	MEAN	SQUARE
X(-9.,+9.)	375	786	392	170	215	51	8	3	.175E-04	.876E-06	.175E-04	.729E-06	
Y(-1.,+1.)	1258	565	11	18	74	73	1	0	.151E-05	.312E-06			
X(-9.,+9.)	374	942	181	54	189	253	7	0	.175E-05	.547E-06			
Y(-8C,+8C)	1418	581	1	0	0	0	0	0	.554E-06	.253E-06	.549E-06	.253E-06	
X(-80,+80)	302	959	288	50	123	254	24	0	.212E-05	.543E-06			
Y(-1.,+1.)	1387	589	2	4	8	10	0	0	.106E-05	.262E-06	.209E-05	.279E-06	
X(-80,+80)	462	1152	169	17	71	117	12	0	.196E-05	.456E-06			
Y(-80,+80)	1390	610	0	0	0	0	0	0	.505E-06	.257E-06	.505E-06	.257E-06	

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(-9.,+9.),	
Y(-1.,+1.)	138.0
UNIFCRM IN X(-80,+80),	
Y(-80,+80)	136.0

ERRCR RETURN IHE211I FROM IHEATH TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDATANH COMPLEX DOUBLE HYPERBOLIC ARCTANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERRCR IN THE LAST HEXADECIMAL DIGIT								COMPONENT			COMPLEX	
									MAX	ROOT	MAX	ROOT	
	0	1	2	3	3BT	4BT	5BT	OTHER	REL	MEAN	REL	MEAN	SQUARE
X(-9.,+9.)	391	671	412	231	212	50	26	7	.151E-13	.402E-15			
Y(-1.,+1.)	1303	509	28	29	81	48	2	0	.116E-14	.865E-16	.128E-13	.328E-15	
X(-80,+80)	797	828	135	64	112	56	8	0	.417E-15	.946E-16			
Y(-80,+80)	1474	526	0	0	0	0	0	0	.152E-15	.731E-16	.152E-15	.731E-16	

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(-9.,+9.),	
Y(-1.,+1.)	218.0
UNIFCRM IN X(-80,+80),	
Y(-80,+80)	220.0

ERRCR RETURN IHE211I FROM IHEATZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CABS COMPLEX ABSOLUTE VALUE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	-4BT	-3BT	-3	-2	-1	0	1	OTHER		
THETA(0,2PI)										
R(0,1)	111	428	196	296	385	324	139	121	.156E-05	.412E-06
THETA(0,2PI)										
R(0,-)	60	241	135	268	721	411	89	75	.162E-05	.531E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN THETA(0,2PI), R(0,1)	78.0
UNIFCRM IN THETA(0,2PI), R(0,-)	82.0

NO ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDABS COMPLEX DOUBLE ABSOLUTE VALUE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	-4BT	-3BT	-3	-2	-1	0	1	OTHER		
THETA(0,2PI)										
R(0,1)	143	448	211	280	388	275	143	112	.440E-15	.999E-16
THETA(0,2PI)										
R(0,-)	17	77	99	318	927	497	43	22	.439E-15	.139E-15

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN THETA(0,2PI), R(0,1)	94.0
UNIFCRM IN THETA(0,2PI), R(0,-)	98.0

NO ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

COS COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	6BT OTHER		
(0.0 ,PI/2)	2860	1948	43	16	50	68	1	2	12	.187E-03
(PI/2, PI)	1885	2926	143	2	15	29	0	0	0	.333E-05
(PI ,2PI)	2297	2517	92	4	39	48	0	0	3	.468E-04
(2PI ,8PI)	2359	2444	93	12	35	51	0	0	6	.110E-02
(8PI ,64PI)	2339	2469	85	14	41	48	1	1	2	.153E-04

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,PI/2)	50.0
X = 0.7854	54.0
X = 5.0	54.0
X = 20.0	54.0

ERROR RETURN IHE212I FROM IHESNS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DCOS DOUBLE COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER		
(-PI/2,PI/2)	82	244	205	117	187	133	1	31		.173E-13
(PI/2, -)	85	202	114	93	271	129	21	85		.224E-12

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (-PI/2,PI/2)	84.0
UNIFCRM IN (PI/2, -)	80.0

ERROR RETURN IHE203I FROM IHESNL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CCOS COMPLEX COSINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT OTHER				
X(0.0,.35)	1595	2521	400	311	156	16	1	0	.192E-05	.353E-06	
Y(0.0, -)	393	1594	884	680	1220	214	15	0	.252E-05	.632E-06	.192E-05 .354E-06
X(.35, -)	1127	2469	558	325	395	104	13	9	.108E-03	.167E-05	
Y(0.0, -)	1180	2402	552	366	385	89	16	10	.268E-02	.380E-04	.158E-05 .267E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(0.0,.35),	
Y(0.0, -)	204.0
UNIFCRM IN X(.35, -),	
Y(0.0, -)	200.0

ERRCR RETURN IHE212I FROM IHESNW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDCOS COMPLEX DOUBLE COSINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT OTHER				
X(0.0,.35)	443	948	261	56	211	80	1	0	.440E-15	.120E-15	
Y(0.0, -)	117	222	193	170	397	399	248	254	.259E-11	.740E-13	.443E-15 .138E-15
X(.35, -)	82	165	114	87	223	268	256	805	.103E-11	.461E-13	
Y(0.0, -)	95	160	142	50	231	239	248	795	.689E-11	.188E-12	.947E-14 .211E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(0.0,.35),	
Y(0.0, -)	298.0
UNIFCRM IN X(.35, -),	
Y(0.0, -)	302.0

ERRCR RETURN IHE203I FROM IHESNZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

COSH HYPERBOLIC COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT									MAX REL ERROR	ROOT MEAN SQUARE
	-4	-3	-2	-1	0	1	2	3	4		
(0.0 , 1.0)	0	0	0	1037	1934	29	0	0	0	.885E-06	.402E-06
(1.0 ,12.88)	48	83	98	658	1755	168	81	75	34	.101E-05	.204E-06
(12.88, -)	37	100	93	625	1823	91	85	100	46	.675E-06	.182E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0 , 1.0)	88.0
UNIFCRM IN (1.0 ,12.88)	84.0
UNIFCRM IN (12.88, -)	84.0
X = 0.20	84.0
X = 8.94	84.0
X = 100.0	84.0

ERROR RETURN IHE300I FROM IHESHS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DCOSH DOUBLE HYPERBOLIC COSINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	-4BT	-3BT	-3	-2	-1	0	1		
(0.0,0.35)	0	0	0	0	561	439	0	.222E-15	.163E-15
(0.35,19.3)	105	96	13	14	512	260	0	.367E-15	.114E-15
(19.3, -)	8	137	30	27	299	494	5	.218E-15	.748E-16

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,0.35)	110.0
UNIFCRM IN (0.35,19.3)	118.0
UNIFCRM IN (19.3, -)	112.0
X = 0.20	116.0
X = 10.0	112.0
X = 50.0	116.0

ERROR RETURN IHE300I FROM IHESHL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CCOSH COMPLEX HYPERBOLIC COSINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							COMPONENT		COMPLEX	
	0	1	2	3	3BT	4BT	5BT OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
X(-10, 10)	144	899	383	113	292	158	9	2	.339E-03	.762E-05	
Y(-1., 1.)	313	912	255	155	291	67	3	4	.230E-03	.538E-05	.179E-05 .556E-06
X(-80, 80)	472	938	242	115	194	36	3	0	.521E-05	.483E-06	
Y(-80, 80)	455	963	223	125	189	37	4	4	.174E-04	.731E-06	.186E-05 .387E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(-10, 10),
 Y(-1., 1.) 208.0
 UNIFCRM IN X(-80, 80),
 Y(-80, 80) 204.0

ERRCR RETURN IHE2121 FROM IHESNW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDCOSH COMPLEX DOUBLE HYPERBOLIC COSINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERRCR IN THE LAST HEXADECIMAL DIGIT							COMPONENT		COMPLEX	
	0	1	2	3	3BT	4BT	5BT OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
X(-10,10.)	133	380	322	100	305	300	148	312	.159E-11	.373E-13	
Y(-20,20.)	175	416	178	113	291	287	143	397	.150E-11	.395E-13	.202E-13 .110E-14
X(-80,80.)	110	246	137	83	259	276	264	625	.249E-11	.760E-13	
Y(-80,80.)	116	268	135	103	251	254	274	599	.370E-11	.840E-13	.970E-14 .134E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(-10, 10),
 Y(-1., 1.) 342.0
 UNIFCRM IN X(-80, 80),
 Y(-80, 80) 338.0

ERRCR RETURN IHE2031 FROM IHESNZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

ERF ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT				MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3		
(-3.919, -2.04)	967	1033	0	0	.597E-07	.346E-07
(-2.04,-1.317)	1080	920	0	0	.694E-07	.335E-07
(-1.317, 0.0)	1179	796	25	0	.752E-06	.991E-07
(0.0 , 1.317)	1178	794	28	0	.848E-06	.101E-06
(1.317, 2.04)	1040	960	0	0	.684E-07	.340E-07
(2.04 ,3.9192)	958	1042	0	0	.595E-07	.348E-07

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X =-1.10	68.0
X = 1.10	66.0
X = 1.80	68.0
X = 2.60	116.0
X = 5.10	28.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DERF DOUBLE ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT				MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3		
(0.0 ,1.317)	1138	836	26	0	.217E-15	.278E-16
(1.317, 2.04)	1097	903	0	0	.148E-16	.953E-17
(2.04 ,6.092)	1050	950	0	0	.139E-16	.967E-17

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X = 0.80	96.0
X = 1.60	134.0
X = 3.20	206.0
X = 6.40	22.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

ERFC COMPLEMENTARY ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER		
(-13.3,-1.32)	1801	199	0	0	0	0	0	0	.450E-06	.121E-06
(-1.32, 0.0)	1095	905	0	0	0	0	0	0	.916E-06	.345E-06
(0.0 ,1.317)	1154	818	28	0	0	0	0	0	.114E-05	.245E-06
(1.317, 2.04)	706	882	326	53	33	0	0	0	.179E-05	.350E-06
(2.04,3.919)	567	639	306	171	261	56	0	0	.105E-05	.368E-06
(3.919,13.31)	38	74	55	60	183	338	433	819	.154E-04	.828E-05

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X =-5.10	28.0
X =-1.10	72.0
X = 1.10	68.0
X = 1.80	68.0
X = 3.80	116.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CERFC DOUBLE COMPLEMENTARY ERROR FUNCTION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	6BT		
(-6.0,-1.32)	1075	925	0	0	0	0	0	0	.114E-15	.756E-16
(-1.32, 0.0)	1067	933	0	0	0	0	0	0	.222E-15	.101E-15
(0.0 ,1.317)	1128	846	26	0	0	0	0	0	.354E-15	.646E-16
(1.317, 2.04)	761	978	181	50	30	0	0	0	.431E-15	.726E-16
(2.04,3.919)	533	668	303	203	269	24	0	0	.349E-15	.923E-16
(3.919,13.31)	38	96	58	46	192	371	408	367	.333E-14	.182E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
X =-0.80	104.0
X = 0.80	132.0
X = 1.60	206.0
X = 3.20	210.0
X =14.30	106.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

EXP EXPONENTIAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT			MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2		
(C.0 , 0.3466)	1984	16	0	.473E-06	.234E-06
(C.3466, 0.6931)	1992	8	0	.341E-06	.166E-06
(0.6931, 1.3863)	1983	17	0	.228E-06	.100E-06
(1.3863, 2.7726)	1932	68	0	.125E-06	.398E-07
(2.7726, 5.5452)	1955	45	0	.439E-06	.119E-06
(5.5452,11.0904)	1967	33	0	.458E-06	.115E-06
(11.0904,22.1807)	1949	51	0	.455E-06	.117E-06
(22.1807,44.3614)	1962	38	0	.464E-06	.117E-06
(44.3614,88.7229)	1951	49	0	.428E-06	.119E-06
(88.7229,174.673)	1949	51	0	.438E-06	.121E-06
(-174.67,-100.0)	1960	40	0	.470E-06	.117E-06
(-100.0 ,-10.0)	1949	51	0	.438E-06	.113E-06
(-10.0 , -1.0)	1958	42	0	.453E-06	.117E-06
(-1.0 , 0.0)	1910	90	0	.827E-07	.315E-07

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,1.0)	58.0

ERROR RETURN IHE300I FROM IHEEXS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DEXP DOUBLE EXPONENTIAL

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT				MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3		
(0.0 ,0.322)	1123	877	0	0	.222E-15	.125E-15
(0.322,0.664)	1101	859	0	0	.159E-15	.917E-16
(0.664,1.328)	1212	788	0	0	.114E-15	.533E-16
(1.328,2.656)	1374	626	0	0	.588E-16	.189E-16
(2.656,5.313)	1255	745	0	0	.222E-15	.613E-16
(5.313,10.62)	1247	753	0	0	.222E-15	.614E-16
(10.62,21.25)	1252	748	0	0	.222E-15	.618E-16
(21.25,42.50)	1292	708	0	0	.222E-15	.594E-16
(42.50,85.00)	1249	751	0	0	.221E-15	.615E-16
(85.00,170.0)	1299	701	0	0	.222E-15	.623E-16
(0.0 ,+332)	1214	786	0	0	.193E-16	.102E-16
(-332,-664)	1467	533	0	0	.270E-16	.120E-16
(-664,-1.33)	1389	611	0	0	.524E-16	.210E-16
(-1.33,-2.66)	1204	756	0	0	.197E-15	.760E-16
(-2.66,-5.31)	1301	659	0	0	.222E-15	.548E-16
(-5.31,-10.6)	1306	654	0	0	.222E-15	.595E-16
(-10.6,-21.2)	1289	710	1	0	.222E-15	.582E-16
(-21.2,-42.5)	1256	743	1	0	.222E-15	.640E-16
(-42.5,-85.0)	1269	730	1	0	.222E-15	.616E-16
(-85.0,-170.)	1275	725	0	0	.222E-15	.581E-16

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (-85,170)	78.0

ERROR RETURN IHE300I FROM IHEEXL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CEXP COMPLEX EXPONENTIAL

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							COMPONENT		COMPLEX		
	0	1	2	3	3BT	4BT	5BT	OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
X(0.0, -)	816	888	125	65	89	8	7	2	.171E-04	.541E-06		
Y(0.0,PI)	765	929	131	76	85	9	1	4	.297E-04	.932E-06	.110E-05	.262E-06
X(0.0, -)	840	855	108	64	121	7	2	3	.120E-04	.501E-06		
Y(PI, -)	757	874	169	78	101	16	0	5	.163E-03	.367E-05	.120E-05	.264E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(0.0, -), Y(0.0,PI)	174.0
UNIFCRM IN X(0.0, -), Y(PI, -)	174.0

ERROR RETURN IHE300I FROM IHEEXW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDEXP COMPLEX DOUBLE EXPONENTIAL

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							COMPONENT		COMPLEX		
	0	1	2	3	3BT	4BT	5BT	OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
X(0.0, -)	382	666	256	120	226	159	101	90	.466E-12	.113E-13		
Y(0.0,PI)	448	814	245	128	239	75	25	26	.817E-13	.192E-14	.434E-15	.170E-15
X(0.0, -)	370	547	208	111	219	173	132	240	.118E-11	.293E-13		
Y(PI, -)	308	514	200	130	222	192	154	280	.352E-11	.818E-13	.310E-12	.135E-13

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(0.0, -), Y(0.0,PI)	278.0
UNIFCRM IN X(0.0, -), Y(PI, -)	282.0

ERROR RETURN IHE300I FROM IHEEXZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

POWER EXPONENTIATION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER		
X(2**- 4,2** 4) Y(- 4, 4)	603	414	376	288	200	100	19	0	0	0	.373E-05	.105E-05
X(2**-16,2**16) Y(-16,16)	156	182	198	220	299	358	286	222	77	2	.248E-04	.754E-05
X(2**-32,2**32) Y(- 8, 8)	150	195	183	215	250	285	254	202	129	137	.115E-03	.216E-04
X(2**-64,2**64) Y(- 4, 4)	114	164	177	166	201	256	288	297	207	130	.601E-04	.175E-04
X(2**- 8,2** 8) Y(-32,32)	152	157	215	187	221	331	275	273	161	28	.369E-04	.108E-04
X(2**- 4,2** 4) Y(-64,64)	166	187	179	189	194	247	310	235	178	115	.609E-04	.164E-04

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.
X,Y UNIFORM IN (0,1) MICROSECS.
 138.0

ERROR RETURN IHE209I FROM IHEXXS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DPOWER DOUBLE EXPONENTIATION

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2BT	3BT	4BT	5BT	6BT	7BT	8BT	OTHER		
X(2**- 4,2** 4) Y(- 4, 4)	328	461	450	309	220	185	45	2	0	0	.125E-14	.348E-15
X(2**-16,2**16) Y(-16,16)	84	152	169	198	301	355	297	264	162	18	.732E-14	.225E-14
X(2**-32,2**32) Y(- 8, 8)	109	128	192	168	254	262	230	225	194	238	.505E-13	.954E-14
X(2**-64,2**64) Y(- 4, 4)	94	121	146	144	167	195	296	312	258	267	.255E-13	.677E-14
X(2**- 8,2** 8) Y(-32,32)	47	100	127	144	214	349	378	328	226	87	.132E-13	.365E-14
X(2**- 4,2** 4) Y(-64,64)	59	108	122	163	241	277	294	295	249	192	.185E-13	.541E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.
X,Y UNIFORM IN (0,1) MICROSECS.
 192.0

ERROR RETURN IHE209I FROM IHEXXL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CPOWER COMPLEX EXPONENTIATION

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS							COMPONENT		COMPLEX	
	0	1	2	3	3BT	4BT	5BT OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
XR(-4, 4)											
XI(-32,32)	37	66	61	66	216	364	526 3664	.135E+00	.200E-02		
YI(-32,32)											
YI(-4, 4)	41	48	70	70	204	364	495 3708	.342E-01	.712E-03	.784E-04	.296E-04
XR(-32,32)											
XI(-4, 4)	34	52	70	59	223	362	525 3675	.965E+01	.136E+00		
YR(-4, 4)											
YI(-32,32)	32	68	58	64	237	328	569 3644	.466E-01	.794E-03	.809E-04	.295E-04

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN XR(-4,4), XI(-32,32), UNIFCRM IN YR(-32,32), YI(-4,4)	352.0
UNIFCRM IN XR(-32,32), XI(-4,4)	
UNIFCRM IN YR(-4,4), YI(-32,32)	354.0

ERROR RETURN IHE210I FROM IHEXXW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CPOWER COMPLEX DOUBLE EXPONENTIATION

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS							COMPONENT		COMPLEX	
	0	1	2	3	3BT	4BT	5BT OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
XR(-4, 4)											
XI(-15,15)	31	33	42	51	169	244	302 1128	.719E-12	.378E-13		
YR(-15,15)											
YI(-4, 4)	26	46	49	39	169	231	294 1146	.137E-11	.522E-13	.142E-13	.437E-14
XI(-15,15)											
XI(-4, 4)	27	62	45	36	171	236	279 1144	.124E-10	.287E-12		
YR(-4, 4)											
YI(-15,15)	30	58	53	59	185	226	309 1080	.196E-11	.688E-13	.134E-13	.467E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN XR(-4,4), XI(-15,15), UNIFCRM IN YR(-15,15), YI(-4,4)	564.0
UNIFCRM IN XR(-15,15), XI(-4,4)	
UNIFCRM IN YR(-4,4), YI(-15,15)	566.0

ERROR RETURN IHE210I FROM IHEXXZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

SIN SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	6BT OTHER		
(0.0 ,PI/2)	1892	2935	134	5	14	20	0	0	.138E-05	.194E-06
(PI/2, PI)	2842	1933	48	18	73	73	1	2	.426E-03	.641E-05
(PI ,2PI)	2366	2441	94	8	35	52	1	1	.139E-04	.358E-06
(2PI ,8PI)	2333	2489	85	9	34	42	0	2	.581E-03	.828E-05
(8PI ,64PI)	2299	2504	83	19	43	47	2	0	.296E-04	.609E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,PI/2)	52.0
X = 0.7854	50.0
X = 5.0	50.0
X = 20.0	50.0

ERROR RETURN IHE2121 FROM IHESNS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DSIN DOUBLE SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER		
(-PI/2,PI/2)	284	497	120	53	38	8	0	0	.385E-15	.513E-16
(PI/2, -)	95	203	133	97	256	121	15	80	.117E-12	.629E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (-PI/2,PI/2)	80.0
UNIFCRM IN (PI/2, -)	84.0

ERROR RETURN IHE2031 FROM IHESNL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CSIN COMPLEX SINE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS								COMPONENT		COMPLEX	
	ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER				
X(0.0,.35)	385	1591	872	663	1241	231	17	0	.252E-05	.645E-06		
Y(0.0, -)	1633	2527	353	317	156	14	0	0	.148E-05	.336E-06	.976E-06	.719E-07
X(.35, -)	1141	2407	572	357	392	104	17	10	.268E-02	.379E-04		
Y(0.0, -)	1148	2492	524	334	389	93	11	9	.108E-03	.167E-05	.113E-05	.257E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(0.0,.35),
Y(0.0, -) 200.0

UNIFCRM IN X(.35, -),
Y(0.0, -) 202.0

ERROR RETURN IHE2121 FROM IHESNW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDSIN, COMPLEX DOUBLE SINE

RANGE	ACCURACY FOR RANDOM COMPLEX ARGUMENTS								COMPONENT		COMPLEX	
	ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER				
X(0.0,.35)	41	483	388	272	615	174	26	1	.706E-15	.196E-15		
Y(0.0, -)	502	1009	142	64	207	76	0	0	.435E-15	.110E-15	.425E-15	.113E-15
X(.35, -)	119	239	136	90	269	250	260	637	.378E-11	.110E-12		
Y(0.0, -)	121	206	127	106	255	238	269	678	.832E-12	.339E-13	.178E-13	.142E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(0.0,.35),
Y(0.0, -) 300.0

UNIFCRM IN X(.35, -),
Y(0.0, -) 300.0

ERROR RETURN IHE2031 FROM IHESNZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

SINH HYPERBOLIC SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	-4	-3	-2	-1	0	1	2	3	4			
(0.0 , 1.0)	0	0	0	875	2041	80	0	0	0	.847E-06	.195E-06	
(1.0 ,12.88)	35	82	94	738	1701	130	94	76	50	.992E-06	.209E-06	
(12.88, -)	37	100	93	625	1823	91	85	100	46	.675E-06	.182E-06	

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0 , 1.0)	36.0
UNIFCRM IN (1.0 ,12.88)	38.0
UNIFCRM IN (12.88, -)	90.0
X = 0.20	36.0
X = 8.94	90.0
X = 100.0	80.0

ERROR RETURN IHE300I FROM IHESHIS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DSINH DOUBLE HYPERBOLIC SINE

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT										MAX REL ERROR	ROOT MEAN SQUARE
	-4BT	-3BT	-3	-2	-1	0	1					
(0.0,0.35)	0	0	0	0	545	455	0	.221E-15	.678E-16			
(0.35,19.3)	104	100	11	11	434	335	5	.335E-15	.107E-15			
(19.3, -)	8	135	32	27	297	496	5	.218E-15	.745E-16			

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0,0.35)	62.0
UNIFCRM IN (0.35,19.3)	116.0
UNIFCRM IN (19.3, -)	120.0
X = 0.20	60.0
X = 10.0	116.0
X = 50.0	118.0

ERROR RETURN IHE300I FROM IHESHL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CSINH COMPLEX HYPERBOLIC SINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							COMPONENT		COMPLEX	
	0	1	2	3	3BT	4BT	5BT OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
X(-10, 10)	399	1073	180	80	203	64	1	0	.160E-05	.435E-06	
Y(-1., 1.)	115	706	355	144	420	233	27	0	.241E-05	.741E-06	.183E-05
X(-80, 80)	430	989	208	145	186	36	4	2	.529E-05	.475E-06	
Y(-80, 80)	443	922	245	144	193	45	4	4	.172E-03	.404E-05	.173E-05
											.390E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
UNIFCRM IN X(-10, 10),
Y(-1., 1.) 200.0
UNIFCRM IN X(-80, 80),
Y(-80, 80) 204.0

ERRCR RETURN IHE212I FROM IHESNW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

COSINH COMPLEX DOUBLE HYPERBOLIC SINE

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							COMPONENT		COMPLEX	
	0	1	2	3	3BT	4BT	5BT OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
X(-10,10.)	471	844	168	79	256	181	1	0	.443E-15	.123E-15	
Y(-1.,1.0)	43	552	326	160	489	385	44	1	.641E-15	.214E-15	.434E-15
X(-80,80.)	126	228	132	89	273	255	263	634	.121E-11	.346E-13	
Y(-80,80.)	110	243	136	104	258	292	254	603	.386E-11	.103E-12	.354E-14
											.135E-14

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
UNIFCRM IN X(-10, 10),
Y(-1., 1.) 336.0
UNIFCRM IN X(-80, 80),
Y(-80, 80) 340.0

ERRCR RETURN IHE203I FROM IHESNZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

SQRT SQUARE ROOT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT			MAX REL ERROR	ROOT MEAN SQUARE
	-1	0	1		
(0.0, .999)	0	9450	550	.420E-06	.406E-07
(0.06, 1.0)	0	9454	546	.113E-06	.303E-07

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
UNIFCRM IN (0.07,5.4) 40.0

ERROR RETURN IHE200I FROM IHESQS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DSQRT DOUBLE SQUARE ROOT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT			MAX REL ERROR	ROOT MEAN SQUARE
	-1	0	1		
(0.0, -)	0	1999	1	.291E-16	.650E-18
(.06,1.0)	0	2000	0	0.0	0.0
(1.0-16.)	0	2000	0	0.0	0.0

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.
UNIFCRM IN (0.07,5.4) 58.0

ERROR RETURN IHE200I FROM IHESQL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CSQRT COMPLEX SQUARE ROOT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT										COMPONENT			COMPLEX		
	OTHER	-3BT	-3	-2	-1	0	1	2	3	3BT	OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE	
X(-1.,+1)	11	109	28	33	544	1169	73	13	14	6	0	.894E-06	.218E-06	.890E-06	.176E-06	
Y(-1.,+1)	7	54	20	24	307	1146	329	25	15	63	10	.916E-06	.220E-06	.890E-06	.176E-06	
X(-- ,+1)	1	18	10	70	780	1035	77	8	1	0	0	.821E-06	.158E-06	.821E-06	.158E-06	
Y(-- ,+1)	0	13	3	32	415	1051	434	34	1	17	0	.780E-06	.156E-06	.584E-06	.119E-06	

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

```
UNIFCRM IN X(0,2PI),
  Y(0.0,1.0)      146.0
UNIFCRM IN X(0,2PI),
  Y(1.0, - )     144.0
```

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CCSQRT COMPLEX DOUBLE SQUARE ROOT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT										COMPONENT			COMPLEX		
	OTHER	-3BT	-3	-2	-1	0	1	2	3	3BT	OTHER	MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE	
X(-1.,+1)	1	83	37	143	780	853	89	13	1	0	0	.221E-15	.511E-16	.191E-15	.382E-16	
Y(-1.,+1)	0	40	10	62	484	861	429	73	14	27	0	.218E-15	.461E-16	.191E-15	.382E-16	
X(-- ,+1)	15	140	43	100	530	955	109	40	29	39	0	.246E-15	.769E-16	.222E-15	.766E-16	
Y(-- ,+1)	6	81	35	70	340	940	310	69	44	94	11	.222E-15	.763E-16	.222E-15	.766E-16	

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

```
UNIFCRM IN X(0,2PI),
  Y(0.0,1.0)      176.0
UNIFCRM IN X(0,2PI),
  Y(1.0, - )     176.0
```

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

TAN TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	6BT		
(0.0 ,PI/4)	1857	2496	542	46	4	55	0	0	.166E-05	.277E-06
(PI/4,8PI)	1893	2536	400	72	83	15	0	1	.762E-05	.336E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.	
UNIFCRM IN (0.0,PI/4)	50.0	
X = 0.10	50.0	
X = 0.50	48.0	
X = 2.00	56.0	
X = 10.0	54.0	

ERROR RETURN IHE213I FROM IHETNS TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DTAN DOUBLE TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER		
(0.0 ,PI/4)	1684	2395	764	107	40	10	0	0	.366E-15	.644E-16
(PI/4,8PI)	307	528	383	235	650	716	387	1794	.421E-11	.956E-13

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.	
UNIFCRM IN (PI/4,8PI)	82.0	
X = 0.10	74.0	
X = 0.50	72.0	
X = 2.00	80.0	
X = 10.0	80.0	

ERROR RETURN IHE204I FROM IHETNL TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CTAN COMPLEX TANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER				
X(-1.,+1.)	90	127	65	58	127	110	63	1360	.650E+01	.529E+00		
Y(-9.,+9.)	329	474	219	166	466	291	0	55	.187E-05	.415E-06	.166E-05	.429E-06
X(-1.,+1.)	5	13	17	6	13	15	7	1924	.660E+01	.977E+00		
Y(-80.,+80.)	1728	145	23	15	55	29	0	5	.127E-05	.138E-06	.117E-05	.140E-06
X(-80.,+80.)	76	150	66	46	119	102	62	1379	.584E+01	.475E+00		
Y(-9.,+9.)	482	669	141	113	340	182	2	71	.188E-05	.514E-06	.188E-05	.518E-06
X(-80.,+80.)	10	16	8	5	15	9	7	1930	.622E+01	.965E+00		
Y(-80.,+80.)	1769	132	22	10	43	13	0	11	.198E-05	.175E-06	.184E-05	.171E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(-1.,+1.),
 Y(-9.,+9.) 110.0
 UNIFCRM IN X(-80.,+80.),
 Y(-80.,+80) 112.0

ERRCR RETURN IHE213I FROM IHETNW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

COTAN COMPLEX DOUBLE TANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT								MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	0	1	2	3	3BT	4BT	5BT	OTHER				
X(-1.,+1.)	85	133	102	51	155	125	61	1288	.176E-08	.140E-09		
Y(-9.,+9.)	272	459	237	195	540	289	8	0	.931E-15	.116E-15	.611E-15	.114E-15
X(-80.,+80.)	0	1	1	3	11	10	13	1961	.351E+01	.878E+00		
Y(-80.,+80.)	1629	154	33	30	96	47	3	8	.993E-14	.289E-15	.595E-14	.241E-15

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG. MICROSECS.

UNIFCRM IN X(-1.,+1.),
 Y(-9.,+9.) 250.0
 UNIFCRM IN X(-80.,+80.),
 Y(-80.,+80) 248.0

ERRCR RETURN IHE204I FROM IHETNZ TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

TANH HYPERBOLIC TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	-3	-2	-1	0	1	2	3		
(0.0 ,0.55)	0	0	2127	2873	0	0	0	.863E-06	.165E-06
(0.55,9.01)	1	11	101	2510	2359	18	0	.249E-06	.395E-07
(9.01, -)	0	0	0	5000	0	0	0	.296E-07	.112E-08

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0 ,0.55)	18.0
UNIFCRM IN (0.55,9.01)	34.0
UNIFCRM IN (9.01, -)	24.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

DTANH DOUBLE HYPERBOLIC TANGENT

RANGE	ACCURACY FOR RANDOM ARGUMENTS ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE
	-3BT	-3	-2	-1	0	1	2		
(0.0 ,0.55)	0	0	0	806	1190	4	0	.215E-15	.487E-16
(0.55,20.1)	9	9	9	20	984	969	0	.165E-15	.140E-16
(20.1, -)	0	0	0	0	2000	0	0	0.0	0.0

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN (0.0 ,0.55)	56.0
UNIFCRM IN (0.55,20.1)	116.0
UNIFCRM IN (20.1, -)	20.0

NC ERROR RETURN

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CTANH COMPLEX HYPERBOLIC TANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	C	1	2	3	3BT	4BT	5BT OTHER				
X(-9.,+9.)	331	507	224	155	455	279	1	48	.163E-05	.429E-06	
Y(-1.,+1.)	78	116	78	52	148	127	65	1336	.517E+01	.427E+00	.149E-05 .439E-06
X(-9.,+9.)	497	664	151	114	343	166	0	65	.171E-05	.506E-06	
Y(-8C,+80)	68	136	75	65	115	102	79	1360	.673E+01	.414E+00	.171E-05 .515E-06
X(-80,+80)	1788	63	34	21	56	32	0	6	.125E-05	.151E-06	
Y(-1.,+1.)	11	17	9	5	22	15	8	1913	.560E+01	.958E+00	.125E-05 .156E-06
X(-8C,+80)	1804	80	18	20	56	17	0	5	.141E-05	.182E-06	
Y(-80,+80)	9	16	15	7	16	21	7	1909	.352E+01	.943E+00	.140E-05 .184E-06

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(-9.,+9.),	
Y(-1.,+1.)	112.0
UNIFCRM IN X(-80,+80),	
Y(-80,+80)	112.0

ERROR RETURN IHE213I FROM IHETNW TESTED

TEST OF IBM 360 PL/I SUBROUTINE LIBRARY

CDTANH COMPLEX DOUBLE HYPERBOLIC TANGENT

ACCURACY FOR RANDOM COMPLEX ARGUMENTS

COMPONENT

COMPLEX

RANGE	ERROR IN THE LAST HEXADECIMAL DIGIT							MAX REL ERROR	ROOT MEAN SQUARE	MAX REL ERROR	ROOT MEAN SQUARE
	C	1	2	3	3BT	4BT	5BT OTHER				
X(-9.,+9.)	219	498	222	202	539	302	18	0	.849E-15	.120E-15	
Y(-1.,+1.)	76	135	76	63	159	127	56	1308	.181E-08	.130E-09	.611E-15 .120E-15
X(-8C,+80)	1609	172	34	24	87	59	8	7	.588E-14	.238E-15	
Y(-8C,+80)	0	4	2	2	4	13	13	1962	.431E+01	.875E+00	.536E-14 .247E-15

AVERAGE EXECUTION TIMES ON IBM 360/75 EXCLUDING JUMPS

ARG.	MICROSECS.
UNIFCRM IN X(-9.,+9.),	
Y(-1.,+1.)	204.0
UNIFCRM IN X(-80,+80),	
Y(-80,+80)	202.0

ERROR RETURN IHE204I FROM IHETNZ TESTED

APPENDIX C

Decimal Equivalents of Selected
Inverse Powers of 2

<u>n</u>	<u>2^{-n}</u>	<u>n</u>	<u>2^{-n}</u>
18	3.815 E-06	50	8.882 E-16
19	1.907 E-06	51	4.441 E-16
20	9.537 E-07	52	2.220 E-16
21	4.768 E-07	53	1.110 E-16
22	2.384 E-07	54	5.551 E-17
23	1.192 E-07	55	2.776 E-17
24	5.960 E-08	56	1.388 E-17
25	2.980 E-08	57	6.939 E-18

APPENDIX D
Typical Test Programs

SAMPLE FORTRAN-IV LIBRARY SUBROUTINE TEST PROGRAM

```

C      TEST PROGRAM FOR SINGLE PRECISION -ALOG- FUNCTION
C          NANCY CLARK
C      DIMENSION X(2000), K(19), A(25), IZ(2)
C      EQUIVALENCE (Y,IY), (DZ,IZ)
C      DOUBLE PRECISION DX,DY,DZ
C      COMMON K
C      --- THE DATA A(I) ARRAY DEFINES TEST INTERVALS ---
C      DATA A(1)/0.0/, A(2)/.0625/, A(3)/.125/, A(4)/.25/
C      X, A(5)/.5/, A(6)/.6065/, A(7)/.779/, A(8)/.8825/
C      X, A(9)/.9395/, A(10)/1.0/, A(11)/1.065/, A(12)
C      X /1.134/, A(13)/1.285/, A(14)/1.65/, A(15)/2./,
C      X A(16)/4./, A(17)/8./, A(18)/16./, A(19)/256./
C      --- FOLLOWING LOOP NECESSARY FOR COMPATIBILITY OF
C      RESULTS WITH EARLIER TEST ON THIS FUNCTION ---
C      DO 6210 I=1,18000
6210 X(1)=RANF(-1)
C      --- READ IN TITLE AND PRINT TITLE AND HEADINGS ---
C      READ 1
      PRINT 1
      PRINT 66
      PRINT 63
      PRINT 64
C      -- FORM THE ARRAY OF RANDOM NUMBERS TO BE USED ---
DO 6220 I=1,2000
6220 X(I)=RANF(-1)
C      --- SET UP FOR TEST OF EACH INTERVAL SPECIFIED ---
DO 6250 L=1,18
CON1=A(L)
CON2=A(L+1)-A(L)
RMS=0
REM=0
DO 6230 I=1,11
6230 K(I)=0
C      --- PERFORM ACCURACY TEST FOR A TEST INTERVAL ---
DO 6240 I=1,2000'
Y=CON1+CON2*X(I)
C      --- COMPUTE DOUBLE AND SINGLE PRECISION LOGS ---
DY=Y
DZ=DLOG(DY)
Y=ALOG (Y)
DY=Y
C      --- COMPUTE AND TABULATE ERRORS IN LOGARITHM ---
RE=DABS((DY-DZ)/DZ)
IF(RE.GT.REM) REM=RE
RMS=RMS+RE*RE
J=IY-IZ(1)
IF(IZ(2).LT.0) J=J-1
IF(IZ(1).LT.0) J=-J
IF(J.GT.5) J=5
IF(J.LT.-5) J=-5
6240 K(J+6)=K(J+6)+1
RMS=SQRT(RMS/2000.)

```

```

C    --- PRINT RESULTS OF ERROR TEST ON AN INTERVAL ---
6250 PRINT 65, A(L),A(L+1),(K(I),I=1,11),REM,RMS
C    --- TEST ERROR RETURN WITH AN ILLEGAL ARGUMENT ---
PRINT 12
Y=ALOG(0.)
Y=ALOG(-1.)
RETURN
1 FORMAT( 72H
X
      )
63 FORMAT(-OACCURACY FOR RANDOM ARGUMENTS--//10X
X -INTERVAL-19X-ERROR IN LAST HEXADECIMAL DIGIT-)
64 FORMAT(23X-OTHER -4   -3   -2   -1   0   1   2-
X,- 3   4   OTHER MRE          RMS-)
65 FORMAT(1X,F9.4,- TO-F9.4,1X,3I4,2I5,I6,2I5,3I4,
X 2X,2E10.3)
12 FORMAT(//-- TEST OF ERROR RETURN-)
66 FORMAT(/35X-ALOG-6X,A8)
END

```

SAMPLE PL/I LIBRARY SUBROUTINE TEST PROGRAM

```

C    TEST OF IHEATW - ATAN(COMPLEX/SHORT)
C
COMPLEX*16 DFA,DFS,DXS,DCATN
COMPLEX*8 FA,XA
REAL*8 MRER,RMSR,MREI,RMSI,MRET,RMST,TC,CS(2),CL(2),RANGEX(2),RANG
XEY(2)
DIMENSION X(2,2000),Y(2,2000),K(2,65),T(4),A(2),IA(4),IS(4)
EQUIVALENCE (DFA,CS(1),IA(1)),(DFS,CL(1),IS(1)),(XA,A(1))
DATA RANGEX(1)/-(-1.,1.)-/ ,RANGEX(2)/(-80,80)-
DATA RANGEY(1)/(-9.,9.)-/ ,RANGEY(2)/(-80,80)-
EXTERNAL DUMCS
INDEX=7
C
C    PRINT HEADING
C
READ 1100
PRINT 1100
PRINT 1101
PRINT 1102
PRINT 1103
C
C    INITIALIZATION AND ARGUMENT GENERATION
C
DFA=(0.000,0.000)
DXS=(0.000,0.000)
M=1
DO 1000 I=1,2000
X(1,I)=-1.0+2.0*RANF(-1)
X(2,I)=-80.0+160.0*RANF(-1)

```

```

Y(1,I)=-9.0+18.0*RANF(-1)
1000 Y(2,I)=-80.0+160.0*RANF(-1)
      DO 1250 MR=1,2
      DO 1250 MI=1,2
      DO 1020 I=1,2
      DO 1010 J=1,65
1010 K(I,J)=0
1020 CONTINUE
      MRER=0.000
      RMSR=0.000
      MREI=0.000
      RMSI=0.000
      MRET=0.000
      RMST=0.000
C   FUNCTION CALCULATION AND COMPARISON LOOP
C
      DO 1220 I=1,2000
      A(1)=X(MR,I)
      A(2)=Y(MI,I)
      DXS=XA
      CALL PLICAL(DUMCS,INDEX,XA,FA)
      DFA=FA
      DFS=DCATN(DXS)
C   COMPONENT M.R.E. AND R.M.S. CALCULATION
C
      TC=DABS((CS(1)-CL(1))/CL(1))
      IF(MRER.LT.TC) MRER=TC
      RMSR=RMSR+TC*TC
      TC=DABS((CS(2)-CL(2))/CL(2))
      IF(MREI.LT.TC) MREI=TC
      RMSI=RMSI+TC*TC
C   TOTAL M.R.E. AND R.M.S. CALCULATION
C
      TC=CDABS((DFA-DFS)/DFS)
      IF(MRET.LT.TC) MRET=TC
      RMST=RMST+TC*TC
C   ERROR DISTRIBUTION
C
      DO 1210 N=1,3,2
      J=IA(N)-IS(N)
      IF(J.NE.1048577) GO TO 1050
      IF(IS(N).GE.0) GO TO 1030
      J=0
      GO TO 1070
1030 IF(IA(N).GE.0) GO TO 1040
      J=-1
      GO TO 1070
1040 J=1
      GO TO 1070
1050 IF(IS(N+1).GE.0) GO TO 1060
      J=J-1
1060 IF(IS(N).GE.0) GO TO 1070

```

```

J=-J
1070 IF(IABS(J).LE.4) GO TO 1090
II=J/4
DO 1080 IND=4,7
J=IND
IF(II.LT.0) J=-IND
II=II/2
IF(II.EQ.0) GO TO 1090
1080 CONTINUE
1090 IF(N.EQ.3) GO TO 1200
K(1,J+8)=K(1,J+8)+1
GO TO 1210
1200 K(2,J+8)=K(2,J+8)+1
1210 CONTINUE
1220 CONTINUE
RMSR=DSQRT(RMSR/2000.0D0)
RMSI=DSQRT(RMSI/2000.0D0)
RMST=DSQRT(RMST/2000.0D0)
PRINT 1104,RANGEX(MR),(K(1,I),I=1,15),MRER,RMSR,MRET,RMST
1250 PRINT 1105,RANGEY(MI),(K(2,I),I=1,15),MREI,RMSI
RETURN
1100 FORMAT(72H
X
      )
1101 FORMAT(/35X-IHEATW - ATAN(COMPLEX/SHORT)-/)
1102 FORMAT(-0ACCURACY FOR RANDOM COMPLEX ARGUMENTS--//- RANGE-23X-ERROR
X IN THE LAST HEX DIGIT-/)
1103 FORMAT(16X-LE-6 -5B -4BT -3BT -3 -2 -1 0 1 2 3
X3BT 4BT 5BT GE+6 M.R.E. R.M.S. T.M.R.E. T.R.M.S.-/)
1104 FORMAT(- CATAN X-A8,2I4,11I5,2I4,1X,4E10.3)
1105 FORMAT(- CATAN Y-A8,2I4,11I5,2I4,1X,2E10.3)
1106 END

```

C
C PL/I DEFINING ROUTINE (COMPLEX/SHORT)

```

C
DUMCS    PROC(INDEX,X,Y),
          DCL(X,Y)COMPLEX FLOAT(6),INDEX FIXED BINARY,
          LAB(13)LABEL,
          GO TO LAB(1 INDEX),
LAB(1)    Y=SQRT(X),RETURN,
LAB(2)    Y=EXP(X),RETURN,
LAB(3)    Y=LOG(X),RETURN,
LAB(4)    Y=SIN(X),RETURN,
LAB(6)    Y=TAN(X),RETURN,
LAB(7)    Y=ATAN(X),RETURN,
LAB(8)    Y=X**Y,RETURN,
LAB(9)    Y=ABS(X),RETURN,
LAB(10)   Y=SINH(X),RETURN,
LAB(11)   Y=COSH(X),RETURN,
LAB(12)   Y=TANH(X),RETURN,
LAB(13)   Y=ATANH(X),RETURN,
END DUMCS,
```

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